

2024
SRCMUJ
4th Semester Examination
M. Sc.
Mathematics
MTM - 401
Continuum Mechanics

Full Marks: 40

Time: 2 hours

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

1. Answer any four of the following questions:

4 × 2

- (a) The velocity components in a fluid flow are given by

$$u = 2x^2 + 5z^2, \quad v = 3y^2 + 4z^2, \quad w = -2z(2x + 3y)$$

Show that the flow is possible. Examine whether the motion is rotational or not.

- (b) Define stress quadric. Determine the Cauchy's stress quadratic at a point whose state of stress is

$$[T_{ij}] = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

where a, b, c are all of same sign.

- (c) A two-dimensional flow field is given as $\psi = xy$. Show that the flow is irrotational and hence find the velocity potential and stream lines.

- (d) Evaluate directly strain invariants for the strain tensor

$$[E_{ij}] = \begin{bmatrix} 5 & -1 & -1 \\ -1 & 4 & 0 \\ -1 & 0 & 4 \end{bmatrix}$$

- (e) State Newton's law of viscosity.

- (f) Show that the curves of constant potential and constant stream function cut orthogonally at the point of intersection.

2. Answer any four of the following questions:

4 × 8

- (a) What is strain quadric? Explain the geometric interpretation of infinitesimal strain tensors. 2 + 6

- (b) (i) Show that $\frac{x^2}{a^2} \tan^2 t + \frac{y^2}{b^2} \cot^2 t = 1$ is a possible form of boundary surface of a liquid and find an expression for the normal velocity. 4
(ii) Given the following stress distribution

$$[T_{ij}] = \begin{bmatrix} x_2 & -x_3 & 0 \\ -x_3 & 0 & -x_2 \\ 0 & -x_2 & T \end{bmatrix}$$

Find T such that stress distribution is in equilibrium with body force $\vec{F} = -g\hat{e}_3$. 4

- (c) (i) What do you mean by perfect fluid? State and prove the Kelvin's circulation theorem for perfect fluid. 1+4
(ii) Defining complex potential. Find the complex potential due to a doublet. 1+2

- (d) (i) Find the strain tensor and rotation tensor for small deformation $\vec{u} = \alpha x_1 x_2 (\hat{i} + \hat{j}) + 2\alpha(x_1 + x_2)x_3 \hat{k}$ where α is a constant. 4
(ii) For the displacement field: $u_1 = X_1^2 X_2$, $u_2 = X_2 - X_3^2$, $u_3 = X_2^2 X_3$. Determine the unit relative displacement vector at $P(1, 2, -1)$ with respect to $Q(4, 2, 3)$. 4

- (e) (i) The stress matrix at a point $P(x_i)$ in a material is

$$[T_{ij}] = \begin{bmatrix} x_1 x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{bmatrix}$$

Find the stress vector at the point $Q(1, 0, -1)$ on the surface $x_1 = x_2^2 + x_3^2$. 3

- (ii) Define linear elastic solid. When it is called isotropic? For the isotropic linear elastic material write down the stress-strain relation and hence find the inversion of Hook's law. 1+1+3
(f) (i) Established Navier-Stokes equation of motion for compressible viscous fluid. 5
(ii) Write down the constitutive equations (stress-strain relations) for isotropic, linear elastic material and hence deduce strain-stress relations. 3