2024
SRCMUJ
4th Semester Examination
M. Sc.
Mathematics
MTM – 402

Full Marks: 40 **Time: 2 hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

Unit-I [Fuzzy Mathematics with Applications]

1. Answer any two of the following questions: **2×2**

(a) Suppose $(10, 14, 19)$ is a triangular fuzzy number. Write its membership function and draw its diagram.

(b) State Zadeh's extension principle.

(c) Evaluate $2(5, 6, 8, 12) + 3(-1, 3, 4) - 5[-1,2] + 7$.

(d) Define normal fuzzy set.

2. Answer any two of the following questions: **2×8**

(a) Using Zadeh's extension principle, prove that $[1,\ 3] + [5, 8] = [6, 11]$. 8

(b) Explain Warner's method to solve a fuzzy LPP. Using this convert the following LPP as a crisp LPP.

$$Max\ Z = 4x_1 + 5x_2 + 9x_3 + 11x_4$$

Subject to
$$x_1 + x_2 + x_3 + x_4 \ \leq \widetilde{15}$$
$$7x_1 + 5x_2 + 3x_3 + 2x_4 \ \leq \widetilde{80}$$
$$3x_1 + 5x_2 + 10x_3 + 15x_4 \ \leq \widetilde{100}$$
$$x_1, x_2, x_3, x_4 \geq 0$$

where, the tolerances are $p_1 = 5, p_2 = 40, p_3 = 30$. 4+4

(c) (i) For the following three interval numbers $[1,\ 4]$, $[2,\ 5]$ and $[3,\ 8]$, show that the distributive law does not holds. 4
(ii) Prove De Morgan's laws for fuzzy sets. 4

(d) What is convex fuzzy set? Show that the fuzzy set with the following membership function is neither normal nor convex: 2 + 6

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < 1 \\ 3(x-1)/8 & \text{for } 1 < x \leq 3 \\ (6-x)/4 & \text{for } 3 < x \leq 4 \\ (3x-2)/20 & \text{for } 4 < x \leq 6 \\ 3(7-x)/5 & \text{for } 6 < x \leq 7 \\ 0 & \text{for } x > 7 \end{cases}$$

Unit – 2 [Real Analysis – II]

3. Answer any two of the following questions: **2×2**

(a) State Lusin's theorem. Write down its significance.

(b) Show that Monotone convergence theorem can be proved as a special case of the Fatou's lemma.

(c) Check whether the following function is L-integrable on $[0, 1]$ or not:

$$g(x) = \begin{cases} 1/\sqrt{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(d) For every $\epsilon > 0$ and $f \in L^1(\mu)$, show that $\mu\{x \in X : |f(x)| \geq \epsilon\} \leq 1/\epsilon \int f \ d\mu$.

4. Answer any two of the following questions: **2×8**

(a) (i) Show that a necessary and sufficient condition for a bounded function f defined on a measurable set E with finite measure, to be measurable, is that, for all simple functions ϕ and ψ on E, $\inf\limits_{\psi \geq f} \int_E \psi(x)dx = \sup\limits_{\phi \geq f} \int_E \phi(x)dx$ 6

(ii) Suppose that $\{f_n\}$ is a sequence of non-negative L-measurable functions on $(0, 10)$ such that $f_n(x) \to f(x)$ for almost all $x \in (0, 10)$. Let $F(x) = \int_0^x f\, dm$ and $F_n(x) = \int_0^x f_n\, dm$. Show that $\int_0^{10} (f + F)dm \leq \liminf\limits_{n\to\infty} \int_0^{10} (f_n + F_n)dm$. 2

(b) (i) State and prove monotone convergence theorem. $1 + 4$

(ii) Using Lebesgue dominated convergence theorem, show that for $a > 0$,

$$\int_{-\infty}^{\infty} e^{-x^2} \cos(ax)dx = \sqrt{\pi}\, e^{-a^2/4} \qquad 3$$

(c) (i) Let $f(x)$ be a measurable function over E. Then show that $f(x)$ is integrable over E if and only if $|f|$ is integrable over E. Moreover, if $f(x)$ is integrable, then show that $\left|\int_E f(x)dx\right| \leq \int_E |f(x)|dx$. 4

(ii) State and prove the Fatou's lemma. Give an example to show that strict inequality can occur in Fatou's lemma. 4

(d) (i) State and prove Lebesgue theorem on bounded convergence. 1 + 4

(ii) Using dominated convergence theorem, evaluate

$$\lim_{n\to\infty}\int_0^n (1 + x/n)^n e^{-2x}\, dx \qquad 3$$

justifying any interchange of limits you use.