

2024
SRCMUJ
4th Semester Examination
M. Sc.
Mathematics
MTM - 403

Full Marks: 40

Time: 2 hours

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

Unit-1 [Stochastic Process and Regression]

1. Answer any two of the following questions:

2 × 2

- (a) State Gambler's ruin problem and write its transition matrix.
- (b) Show that the bivariate correlation coefficients r_{12}, r_{13} and r_{23} must satisfy the inequality $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23} \leq 1$.
- (c) Write the main criteria for which a stochastic process called Wiener process.
- (d) Define transient and persistent states. When a persistent state is called null-persistent?

2. Answer any two of the following questions:

2 × 8

- (a) (i) Write the main postulates for Poisson process.
 (ii) Under the above postulates, show that $N(t)$ follows Poisson distribution with mean λt i.e; $p_n(t)$ is given by the Poisson Law, $p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0, 1, 2, \dots$, where $N(t)$ is the number of occurrences of an event E in an interval of time $(0, t)$.
2 + 6
- (b) (i) Write the graphical representation of a Markov Chain.
 (ii) Prove that $r_{1.23...p} = (1 - \frac{|R|}{R_{11}})^{1/2}$, where the symbols have their usual meaning.
 Hence discuss the case if we take three variables.
2 + 6
- (c) (i) Find the differential-difference equation for the birth and death process. **4**
 (ii) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the n^{th} stage of the system and X_0 be the digit entering the first stage (leaving the 0th stage). At each stage, there is a constant probability q that the digits which enter will be transmitted unchanged (i.e., the digit changes when it leaves), $p + q = 1$. Find the one-step transition matrix P , and n -step transition matrix P^n . Also, find P^n when $n \rightarrow \infty$. **4**
- (d) Let $\{X_n, n \geq 0\}$ be a branching process. Show that if $m = E(X_1) = \sum_{k=0}^{\infty} k p_k$ and $\sigma^2 = \text{Var}(X_1)$, then $E(X_n) = m^n$ and **8**

$$\text{Var}(X_n) = \begin{cases} \frac{m^{n-1}(m^n - 1)\sigma^2}{m - 1} & \text{if } m \neq 1 \\ n\sigma^2 & \text{if } m = 1 \end{cases}$$

Unit – II [Graph Theory]

3. Answer any two of the following questions:

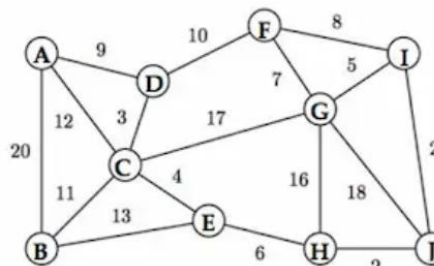
2×2

- (a) Define Peterson Graph and find its chromatic number.
- (b) Which complete graph $K_{m,n}$ are Euler graph?
- (c) Find the maximum number of edges of simple graph with n vertices.
- (d) Is C_6 a bipartite graph? Justify your answer.

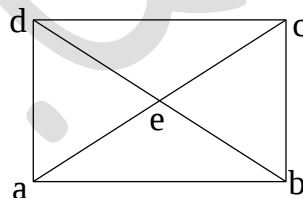
4. Answer any two of the following questions:

2×8

- (a) (i) Define minimally connected graph. Show that a graph is a tree if and only if it is minimally connected. 1 + 2
- (ii) Find a minimum spanning tree for the following graph: 5



- (b) (i) Prove that the maximum number of vertices in a binary tree of depth k is $2^k - 1$.
- (ii) Suppose G is a graph with 1000 vertices and 3000 edges. Is G planar? 4 + 4
- (c) (i) State the decomposition theorem. Use the decomposition theorem to find the chromatic polynomial of the following graph: 2 + 4



- (ii) Find the graph whose adjacency matrix is $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$. 2

- (d) (i) Define intersection graph. 2
- (ii) State the Max-Flow Min-Cut theorem. Find the Max-flow of the following network. 6

