2024
SRCMUJ
4th Semester Examination
M. Sc.
Mathematics
MTM-404 (B)
Special-paper OR: Advanced Optimization-II

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

1. Answer any four questions from the following: **4 × 2**

(a) What is the necessity of constraint qualification related with non-linear programming?

(b) What is multi-objective non-linear programming problem? Give an example of it.

(c) Write one advantage and one disadvantage of geometrical programming problem.

(d) Define bi-matrix game with example.

(e) Define: Nash equilibrium strategy and Nash equilibrium outcome.

(f) Write two important methods for solving stochastic programming problem.

2. **Answer any four questions from the following:** **4 × 8**

(a) (i) Let Γ be an open convex set in $\mathbb{R}^n$. If θ is a convex numerical function on Γ then show that θ is continuous on Γ.
(ii) State and prove strict separation theorem for convex set. 5 + 3

(b) (i) What is differentiable concave function? Give the geometrical interpretation of it.
(ii) State and prove Kuhn Tucker saddle point sufficient optimality theorem. 3 + 5

(c) (i) Define the following terms: Minimization problem, Local minimization problem, Kuhn-Tucker stationary point problem, Fritz-John stationary point problem.
(ii) Using the chance constrained programming technique to find an equivalent deterministic LPP to the following Stochastic programming problem.

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } \sum_{j=1}^{n} a_{ij} x_j \leq b_j, x_j \geq 0, i, j =, 2, \dots, n,$$

where c_j is random variable. 4 + 4

(d) (i) Find the Nash equilibrium solution(s) of the following bi-matrix game (if exists)

$$\begin{bmatrix} (-2,-1) & (1,1) \\ (-1,2) & (-1,-2) \end{bmatrix}$$

(ii) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset \mathbb{R}^n$. Then prove that θ is convex on Γ if and only if $\theta(x^2) - \theta(x^1) \geq \theta(x^1)(x^2 - x^1)$ for each $x^2,\ x^1 \in \Gamma$.

3 + 5

(e) (i) State and prove Motzkin's theorem of alternative.

(ii) Write the formulations of an N-person finite static game in normal form. 4 + 4

(f) Solve the quadratic programming problem using Wolfe's method 8

$$\text{Maximize } Z = 2x_1 + 3x_2 - 2x_1^2$$
$$\text{Subject to} \quad x_1 + 4x_2 \leq 4,$$
$$x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$