

2024
SRCMUJ
M.Sc.
Mathematics
4th Semester Examination
MTM-405(B)
Special Paper-OR: Operational Research II

Full marks: 20

Time: 1 hour

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

1. Answer any two of the following questions: 2×2

- (a) What is transversality condition of Euler's equation in study of calculus of variations?
- (b) What is noiseless Channel? Give an example of such channel matrix.
- (c) Define reliability. Distinguish reliability from probability?
- (d) Explain the Shannon-Fano encoding procedure.

2. Answer any two of the following questions: 2×8

- (a) (i) Define system reliability. Find the reliability of a system with two components of which one is stand-by the components are connected in parallel.
- (ii) An electronic circuit consists of 5 silicon transistors, 3 silicon diodes, 10 composite resistors and 2 ceramic capacitors in series configuration. The hourly failure rate of each component is

for transistors 4×10^{-5}
 for diode 3×10^{-5}
 for resistor 2×10^{-4}
 for capacitor 2×10^{-4}

Calculate the reliability of the circuit for 10 hours when the components follow exponential distribution. 4+4

- (b) Find the stationary path $x = x(t)$ for the functional $J = \int_0^1 [1 + \dot{x}^2] dt$ subject to the boundary condition $x(0) = 0, \dot{x}(0) = 1, x(1) = 1, \dot{x}(1) = 1$. 8

- (c) A source memory has six characters with the following probabilities of transmission:

Character	A	B	C	D	E	F
Probability	1/3	1/4	1/8	1/8	1/12	1/12

Devise the Shannon-Fano encoding procedure to obtain uniquely decodable code to the above message ensemble. What is the average length efficiency and redundancy of the code that you obtain? 8

(d) A transmitter and the receiver have information consisting of three letters. The joint probability for the communication is given below:

$P(x_i, y_j)$	y_1	y_2	y_3
x_1	0.25	0.28	0.05
x_2	0.06	0.12	0.02
x_3	0.04	0.08	0.10

Determine the entropies $H(x)$, $H(y)$ and $H(x/y)$ for this channel.

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