2024
SRCMUJ
4th Semester Examination
M.Sc.
Mathematics
MTM-495B
Special Paper-OR: Lab. (OR methods using MATLAB and LINGO)

Full Marks: 25 **Time: 3 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Here R is your roll number.

Group-A [MATLAB]

1.Answer any one by lottery basis: **1×12**

(a) Write a MATLAB code to estimate the probability of obtaining 8 or more heads if a coin is tossed 10 times**.** Use the Monte Carlo simulation technique for this purpose. The real value is approx. 0.0547.

(b) Write a MATLAB code to solve the following problem using the Dual-Simplex method

$$\begin{aligned}
\text{Minimize}\quad & z = x_1/3 + 5x_2 - x_3 + x_4 + Rx_5 \\
\text{subject to}\quad & x_1 + x_2 + x_3 + x_4 + x_5 \le 10 \\
& -5x_1 - 2x_2 + x_3/5 + x_4 + 3x_5 \le 5 \\
& -x_1 + x_2 - 2x_3 + 5x_4 - x_5 \le 2 \\
& 2x_1 - 4x_2 + 3x_3 - x_4/2 - 2x_5 \le 9 \\
& x_1/2 - x_3 - x_5 = -1 \\
& x_1, x_2, x_3, x_4, x_5 \ge 0
\end{aligned}$$

(c)Write a program in MATLAB to solve the IPP

$$\begin{aligned}
\text{Minimize}\quad & z = -x_1 + 5x_2 - x_3 + Rx_4 + x_5 \\
\text{subject to}\quad & x_1 - 2x_2 + x_3 + x_4 + 3x_5 \le 2 \\
& -x_1 + x_2 - 2x_3 + 5x_4 - x_5 \le 8 \\
& 2x_1 - 4x_2 + 3x_3 - x_4 - 2x_5 \le 7 \\
& x_1 - x_3 - 2x_5 = -5
\end{aligned}$$

where $x_i : i = 1(1)5$ are non-negative integer variables.

(d) In a railway marshalling yard, goods trains arrive at a rate of 45 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of (75-R) minutes. Write a program in MATLAB to find the following.

i) The average number of trains in the system
ii) The average number of trains in the queue

iii) Mean (or expected) waiting time in the queue (excluding service time)
iv) Expected waiting time in the system (including service time)

(e) Write a MATLAB code to solve the following QPP

$$\begin{aligned}
\text{Minimize} \quad & z = x_1^2 + 5x_2^2 - x_3^2 + 4x_1x_2 - x_1x_3 + 3x_3 - x_2 \\
\text{subject to} \quad & -x_1 + x_2 + 2x_3 \le 4 \\
& 3x_1 - 2x_2 + x_3 \le 5 \\
& -x_1 - x_2 + x_3 = 2 \\
& x_1,\ x_2,\ x_3 \ge 0
\end{aligned}$$

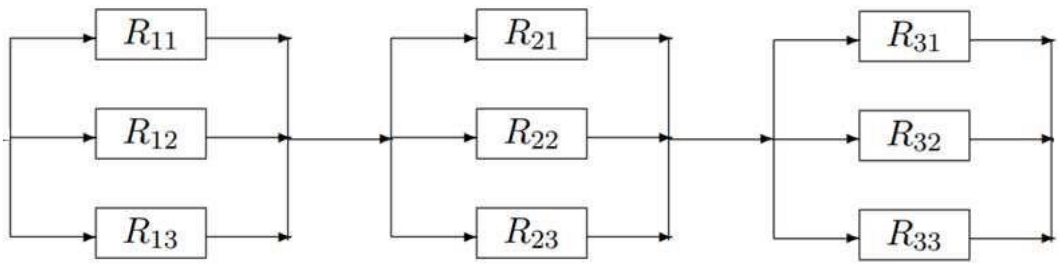
(f)A contractor has to supply 10000-2R (R is the roll number of the student) bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25000 bearings per day. The cost of holding a bearing in stock for one year is Rs. 2, and the set-up cost of a production run is Rs. 1800.
Write a program in MATLAB to find the following.
i) The lot size which will minimize the cost of the system.
ii) The optimal cost.
iii) Cycle time.

(g) Solve the following non-linear programming problem by dynamic programming method using MATLAB:

$$\begin{aligned}
\text{Minimize} \quad & z = x_1^2 + 5x_2^2 - x_3^2 + 4x_1x_2 - x_1x_3 + Rx_4^2 \\
\text{subject to} \quad & 2x_1 + 11x_2 + 2x_3 \le 40 \\
& 13x_1 + 7x_2 + x_3 \ge 25 \\
& x_1,\ x_2,\ x_3,\ x_4 \ge 0
\end{aligned}$$

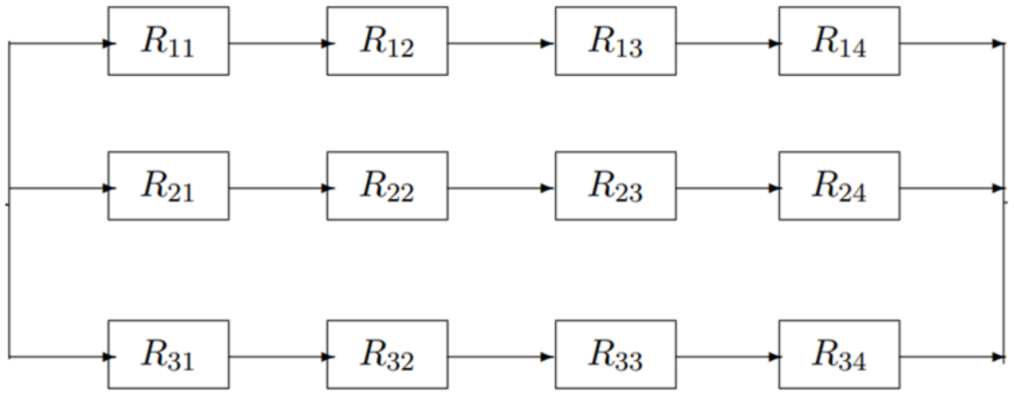
(h)The system is comprised of $570 - R$ transistors, $9215 - 2R$ resistors, and $470 - R$ capacitors connected in series. The failure rates for these components are as follows: transistors have a failure rate of 0.5×10^{-7} per hour, resistors have a failure rate of 0.3×10^{-6} per hour, and capacitors have a failure rate of 0.7×10^{-6} per hour. Find the failure rate of the system and its reliability over duration of 90 hours.

(i) Consider the following diagram of a system of components.



The reliability of the component R_{ij} is given by $R_{ij}(t) = e^{-(i+j)\lambda t}$.
Calculate the reliability of the system for 100 hours for $\lambda = 0.004$.

(j) Consider the following diagram of a system of components.



The reliability of the component R_{ij} is given by $R_{ij}(t) = e^{-(ij)\lambda t}$.
Calculate the reliability of the system for 100 hours for $\lambda = 0.005$.

(k) **NotationsandFormula**

λ:Arrivalrate

μ:Servicerate

P_0: Probabilityof nocustomer inthe system $= \frac{\mu-\lambda}{\mu}$

L_s: Expected(average)numberofunitsinthesystem $= \frac{\lambda}{\mu-\lambda}$

L_q: Expected(average)queuelength $= \frac{\lambda^2}{\mu(\mu-\lambda)}$

W_q: Mean (or expected)waitingtimeinthequeue(excludingservicetime) $= \frac{\lambda}{\mu(\mu-\lambda)}$

W_s: Expected waiting time in the system (including service time) $= \frac{1}{\mu-\lambda}$

Problem: In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day.Assuming that the inter-arrival time follows an exponential distribution and the service timedistribution is also exponential with an average of 36 minutes. Write a program in MATLABtofind thefollowing.

i) Theaveragenumberoftrainsinthesystem
ii) Theaveragenumberoftrainsinthequeue
iii) Mean(or expected)waitingtimeinthequeue(excludingservicetime)
iv) Expectedwaitingtimeinthesystem(includingservicetime)

Group-B [LINGO]

2. Answer any one by lottery basis: **1×8**

(a) Write a code in LINGO to solve the following LPP using Simplex method.

$$
\begin{aligned}
\text{Maximize} \quad & z = Rx_1 - 15x_2 \\
\text{subject to} \quad & x_1 + 2x_2 \le 50 \\
& -5x_1 - x_2 \le 12 \\
& -x_1 + x_2 = 27 \\
& 3x_1 - 2x_2 = -10 \\
& x_1,\ x_2 \ge 0
\end{aligned}
$$

(b) Write the code in LINGO to solve the following problem on Inventory.
An engineering factory consumes 15000 units of a component per year. The ordering, receiving and handling cost are Rs.400 per order while trucking cost is Rs.1500 per order, internet cost Rs. 0.07 per unit per year, Deterioration and obsolesce cost Rs 0.004 per year and storage cost Rs. 1500 per year for 5500 units. Calculate the economic order quantity and minimum average cost.

(c) Write the code in LINGO to find the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}.$$

(d) Write the code in LINGO to solve the following problem on Inventory.

The demand for an item in a company is 1900 units per year. The company can produce the item at a rate of 3200 per month. The cost of one set-up is Rs. 550 and the holding cost of one unit per month is Rs. 0.25. The shortage cost of one unit is Rs. 25 per month. Determine the optimum manufacturing quantity. Also determine the manufacturing time and the time between setup.

(e) Write a code in LINGO to solve the Nash equilibrium strategy and Nash equilibrium outcome of the following bi- matrix game.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ 3 & -5 \end{bmatrix}$$

(f) WriteacodeinLINGO tosolvethefollowingStochasticProgrammingProblem.

A manufacturing firm produces two machine parts using lathes,milling machinesand grinding machines. Themachining times available per week on differentmachines and the profit on machine part are given below. Themachining times required on different machines for each part are not known precisely (as they vary from worker toworker) but are known to follow normal distribution with mean and standard deviations as indicated in the followingtable.

TypeofMachine	Machiningtimerequiredperunit(minutes)				Maximum timeavailableper week(minutes)
	Par tI		Par tI		
	Mean	Mean	Mean	Mean	
Lathes	$\bar{a}_{11}=7$	$\sigma_{a11}=4$	$\bar{a}_{12}=6$	$\sigma_{a12}=3$	$b_1=2310$
Millingmachines	$\bar{a}_{21}=5$	$\sigma_{a21}=2$	$\bar{a}_{22}=17$	$\sigma_{a22}=9$	$b_2=1406$
Grindingmachine	$\bar{a}_{31}=10$	$\sigma_{a31}=6$	$\bar{a}_{32}=7.5$	$\sigma_{a31}=8$	$b_3=3450$

Profitper unit(Rs)	c_1=540	c_2=1000	

DeterminethenumberofmachinepartsIandIItobemanufacturedperweektomaximizetheprofitwithoutexceedingtheavailable machiningtimesmorethanonce in150weeks.

(g) Write the code in LINGO to solve the following problem on Queuing problem.

A telephone exchange has two long distance operators. The telephone company finds that, during the peak load long distance all arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on this call is approximately exponentially distributed with mean length 5 minutes.

(i) What is the probability that a subscriber will have to wait for this long-distance call during the peak hours of the day?

(ii) If the subscriber waits and are serviced in turn, what is the expected waiting time.

(h)Write the code in LINGO to solve the following problem on QPP using Wolfe's modified simplex method.

$$\text{Max } z=2x_1+x_2-x_1^2$$
$$\text{Subject to, } 2x_1+3x_2 \le 6$$
$$2x_1+x_2 \le 4$$
$$x_1, x_2 \ge 0$$

(i) Write a code in LINGO to solve the following Integer Programming Problem using Gomory's cutting plane method.

$$\text{Max } z = 3x_1 - 2x_2 + 5x_3$$
$$\text{Subject to, } 5x_1 + 2x_2 + 7x_3 \le 28$$
$$4x_1 + 5x_2 + 5x_3 \le 30$$
$$x_1, x_2 \ge 0 \text{ and are integer}$$

(j) Write a program in LINGO to solve the LPP using Gomory's Cutting plane method

$$\text{Maximize } Z= 11x_1+4x_2$$
$$\text{Subject to } -x_1+2x_2 \le 4$$
$$5x_1+2x_2 \le 16$$
$$2x_1 - x_2 \le 4$$
$$x_1, x_2 \ge 0 \text{ and are integers}$$

(k) Write a program in LINGO to solve the Geometric Programming

$$\text{Minimize } z= 7x_1x_2^{-1}+3x_2x_3^{-2}+5x_1^{-3}x_2x_3+x_1x_2x_3$$
$$\text{and } x_1,x_2, x_3 \ge 0$$

(l) A system connected in series of 200 transistors, 400 diode, 1050 resistors and 100 capacitors. Failure rate of these components are as follows:

Transistor: $\lambda_t = 0.5\times10^{-6}$ *per hour*

Diode: $\lambda_d = 0.1\times10^{-6}$ *per hour*

Resistor: $\lambda_r = 0.1\times10^{-6}$ *per hour*

Capacitor: $\lambda_c = 2\times0.2\times10^{-6}$ *per hour*

Write a program in LINGO to find the failure rate of the system. Also find the reliability of the system for 100 hours.

Lab note book and Viva-voce **5**