2024
SRCMUJ
2nd Semester Examination
M. Sc.
Mathematics
MTM-203
Numerical Analysis

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Notations have their usual meaning.

1. Answer any four of the following questions: **2 × 4**

(a) Consider the function $f(x) = \frac{x^3}{12} - \frac{x^2}{4} - \frac{x}{3} + \frac{3}{2}$, $1 \le x \le 2$,

$= -\frac{x^3}{12} + \frac{3x^2}{4} - \frac{7x}{3} + \frac{17}{6}$, $2 \le x \le 3$.

Show that $f(x)$ is a cubic spline.

(b) The matrix $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of lower triangular matrix L and upper triangular matrix U. Find properly decomposed L and U matrices respectively.

(c) Obtain the first three orthogonal polynomial $f_n(x)$ on [-1, -1] from linear independent functions $1,\ x,\ x^2$ with weight function $w(x) = 1$.

(d) Define ill-conditioned matrix. Use condition number to show that the matrix

$A = \begin{bmatrix} 1 & 3 \\ 0.33 & 1 \end{bmatrix}$ is ill-conditioned.

(e) Find the weights w_1, w_2, w_3 so that the relation

$$\int_{-1}^{1} f(x)dx = w_1 f\left(-\sqrt{0.6}\right) + w_2 f\left(0\right) + w_3 f\left(\sqrt{0.6}\right)$$

is exact for the functions $f(x) = 1, x, x^2$.

(f) Define predictor and corrector formula in a multi-step method to solve a differential equation. Which formula gives better approximate value and why.

2. Answer any four of the following questions: **8 × 4**

(a) Using Jacobi's method find all the eigen values and corresponding eigen vectors of the

symmetric matrix $A = \begin{bmatrix} 2 & 3 & \frac{1}{\sqrt{2}} \\ 3 & 2 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 5 \end{bmatrix}$. 8

(b) The differential equation $\frac{dy}{dx} = x^2 + y^2 - 2$ satisfies the following data:

x	-0.1	0.0	0.1	0.2
y	1.09	1.00	0.89	0.76

Use Milne's predictor-corrector formula to obtain the value of $y(0.3)$. 8

(c) (i) Using Lagrange's bivariate interpolation method, find the value of $f(2,2)$ from the following data, $f(0,0) = 1,\ f(1,0) = 2,\ f(3,0) = 4,$
$f(0,1) = 3,\ f(1,1) = 5,\ f(3,1) = 15.$

(ii) Write down the expressions of zeros of Chebyshev polynomial of degree n. 6 + 2

(d) (i) Derive the value of $\int_a^b f(x)\,dx$ using Gauss-Chebyshev quadrature formula. 4

(ii) Solve the tri-diagonal system of equation

$$x + y = 3,\ -x + 2y + z = 6,\ 3y + 2z = 12$$ 4

(e) Describe the Crank-Nicolson implicit method to solve the following equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

Subject to the boundary conditions $u(0,t) = f_1(t), u(1,t) = f_2(t)$ and initial condition $u(x,0) = g(x)$. 8

(f) (i) Find the first approximate values at the interior mesh points of the following Dirichlet's problem: $u_{xx} + u_{yy} = 0$. Given that $u(x,0) = 0,\ u(0,y) = 0,\ u(x,1) = 10x,$ $u(1,y) = 10y$.
(The region $x \geq 0, y \leq 1$ be divided into 4×4 squares of sides $h = 0.25$). 4

(ii) Solve Poison's equation $u_{xx} + u_{yy} = 8x^2y^2$ for the square grid as shown below: 4