2024
SRCMUJ
2nd Semester Examination
M. Sc.
Mathematics
MTM-204

Full Marks: 40 **Time: 2 hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

Unit I [Elements of Operations Research]

1. Answer any two of the following questions: **2×2**

(a) What is the difference between lead time and time horizon in inventory control?

(b) Write some service discipline in which customers in the queue will be served.

(c) Arrivals on a telephone booth are considered to be Poisson distribution with an average time of 10 minutes between one arrival and next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes. What is the probability that a person arriving at the booth will to wait?

(d) What is point of inflection?

2. Answer any two of the following questions: **2×8**

(a) Solve using Kuhn-Tucker conditions

$$\text{Maximize } f = 5 + 8x_1 + 12x_2 - 4x_1^2 - 4x_2^2 - 4x_3^2$$
$$\text{Subject to } x_1 + x_2 \leq 1$$
$$2x_1 + 3x_2 \leq 6.$$
8

(b) For the queuing model $(M/M/1)$: $(\infty/FCFS/\infty)$ derive and solve steady state difference equation. Also, find the variance of queue length. 8

(c) Patients arrive at a clinic according to a Poisson distribution at the rate of 50 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour.

(i) Find the effective arrival rate at that clinic.

(ii) What is the probability that an arrival patient will not wait?

(iii) What is the expected waiting time until a patient is discharged from the clinic?

2+3+3

(d) A workshop produces three machine parts A, B, C and the total storage space available is 640 sq. meters. Obtain the optimal lot-size for each item from the following data:

	Items		
	A	B	C
Cost per unit (Rs.)	10.00	15.00	5.00
Storage space required (sq. meter/unit)	0.60	0.80	0.45

Ordering cost (Rs.) (C_3)	100	200	75
No. of units required/year	5000	2000	10,000

The carrying charge on each item is 20% of unit cost.

Unit II [Calculus on R^n]

3. Answer any two of the following questions: **2×2**

(a) Let $f: R^4 \to R^3$ be defined by $f(x, y, z, w) = (x^2y,\ xyz,\ x^2 + y^2 + zw^2)$. Find $f'(a)$ at $a = (1, 2, -1, -2)$

(b) Find the directional derivative of $f(x, y, z) = x^2y^3 - 4xz$ in the direction of $\overrightarrow{v} = (-1,\ 2,\ 0)$.

(c) Find the slope of curve of intersection of the ellipsoid $\frac{x^2}{24} + \frac{y^2}{12} + \frac{z^2}{6} = 1$ made by the plane $y = 1$ at the point $(4,\ 1,\ \sqrt{3/2})$.

(d) Show that the functions $u = x + y - z, v = x - y + z, w = x^2 + y^2 + z^2 - 2yz$ are not independent.

4. Answer any two of the following questions: **2×8**

(a) Find the Jacobian $\frac{\partial(y_1, y_2, \ldots, y_n)}{\partial(x_1, x_2, \ldots, x_n)}$ if $y_1 = 1 - x_1, y_2 = x_1(1 - x_2), y_3 = x_1x_2(1 - x_3), \ldots, y_n = x_1x_2 \ldots x_{n-1}(1 - x_n)$. 8

(b) If $u = \frac{(x^2+y^2)^n}{2n(2n-1)} + xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)^n$. 8

(c) Suppose f maps on open set $E \subset R^n$ into R^m. Then $f \in C'(E)$ if and only if the partial derivatives $D_j f_i$ exists and are continuous on E for $1 \le i \le m, 1 \le j \le n$.

Obtain the Taylor series expansion for the function $f(x, y) = x + 2y + xy - x^2 - y^2$ at $(\frac{4}{3}, \frac{5}{3})$. 4+4

(d) Write the sufficient condition of differentiability of a function of two variables. If $V = f(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$ then prove that

$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = f''(r) + \frac{2}{r}f'(r)$. If $f(r) = r^m$ then find the value of left-hand side. 2+4+2