

2024
SRCMUJ
2nd Semester Examination
M. Sc.
Mathematics
MTM-205
General Topology

Full Marks: 40

Time: 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Notations have their usual meaning.

1. Answer any four of the following questions:

2 × 4

- (a) Determine the closure of the $A = \left\{ \frac{1}{2} \times y ; 0 < y < 1 \right\}$.
- (b) Is the space $\mathbb{R}_l$ second countable? Justify your answer.
- (c) Consider the following subset of the real line $Y = [0, 1] \cup (2, 3)$ in the subspace topology. Examine whether the subsets $[0, 1]$ and $(2, 3)$ are open or closed in Y .
- (d) Define subspace topology. Show that if Y be a subspace of X , U is open in Y and Y is open in X then U is open in X .
- (e) Let X be a topological space such that for each pair of disjoint closed sets A and B in X , there is a real-valued continuous function f on X satisfying $f(x) = 0, \forall x \in A$ and $f(x) = 1, \forall x \in B$. Show that X is normal.
- (f) What is a net in a topological space? Prove that in a topological space X , a subset A of X is closed if and only if no net in A converges to a point of $X - A$.

2. Answer any four of the following questions:

8 × 4

- (a) (i) Define basis of a topological space. Let X be a set and $\mathcal{B}$ be a basis for a topology τ on X . Then show that τ is equal to the collections of all unions of elements of $\mathcal{B}$.
- (ii) Is a normal topological space always regular? Justify your answer.
- (iii) Let $f_n: X \rightarrow Y$ be a sequence of continuous function from the topological space X to the metric space Y . If $\{f_n\}$ converges uniformly to f , then show that f is continuous.

3 + 2 + 3

- (b) (i) Define Frechet compact or B-W compact topological space.

- (ii) Give an example of a second countable B-W compact topological space which is not countably compact. Explain the reason.
- (iii) Prove that a topological space is compact if and only if each family of closed sets in it with finite intersection property has a non-empty intersection. 1 + 3 + 4
- (c) (i) Show that the image of a locally connected topological space under a continuous mapping may not be locally connected.
- (ii) Let X be a subspace of the Euclidean space $\mathbb{R}^2$ defined by $X = \{(0, 0)\} \cup B$, where $B = \{(x, y) ; 0 < x \leq 1 \text{ and } y = \sin \frac{\pi}{x}\}$. Prove that X is connected but not locally connected topological space. 3 + 5
- (d) (i) Prove that a subset of $\mathbb{R}$ is connected if and only if it is an interval.
- (ii) Show that every second countable topological space is Lindeloff. 5 + 3
- (e) (i) Define totally disconnected topological space. Let (X, τ) be a compact Hausdorff topological space which is totally disconnected. Show that it has an open base whose sets are closed.
- (ii) Is the topological space $[0, 1]^n \subseteq \mathbb{R}^n$ compact with respect to usual topology? Justify your answer. 4 + 4
- (f) (i) Define Cartesian product of two topological spaces and its basis.
- (ii) Show that in a Hausdorff topological space, each net converges to at most one point.
- (iii) Let (X, τ) be a topological space and A be a subset of X . Then show that a point s is an accumulation point if and only if there exists a net in $A - \{s\}$ which converges to s . 2 + 3 + 3