2024
SRMUJ
M.Sc.
1st Semester Examination
Mathematics
MTM-102
Complex Analysis

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations used here have their usual meaning.

1. Answer any four of the following questions: **2 × 4**

(a) If $f(z) = \frac{x^3(y-ix)}{x^4-y^4}$ when $z \neq 0$, $f(0) = 0$ then find $\lim_{z\to 0} \frac{f(z)-f(0)}{z-0}$ along the curve $y = x^2$.

(b) Suppose $\text{Im}[f'(z)] = 6x(2y-1)$ and $f(0) = 3-2i, f(1) = 6-5i$,then find $f(1+i)$.

(c) Evaluate $\oint \frac{dz}{z-2}$ around $|z-2| = 4$.

(d) Calculate the sum of all possible residues of the function $f(z) = \frac{z^2}{(z-1)(z^2-5z+6)}$.

(e) Prove that for closed polygons, the sum of the exponents $\frac{\alpha_1}{\pi} - 1$, $\frac{\alpha_2}{\pi} - 1, \ldots\ldots, \frac{\alpha_n}{\pi} - 1$ in Schwarz-Christoffel transformation is equal to -2.

(f) Evaluate $\oint_C \frac{sin^6 z}{z-\frac{\pi}{6}} dz; C: |z| = 1$.

2. Answer any four of the following questions: **8 × 4**

(a) (i) Let $u = x^2 - y^2$ and $v = -\frac{y}{x^2+y^2}$. Is $f(z) = u + iv$ analytic function? Justify your answer. Also, check whether u and v are harmonic or not.

(ii) In the transformation $z = \frac{i-w}{i+w}$, show that half of w-plane given by $v \geq 0$ corresponds to the circle $|z| \leq 1$ in z-plane. 4 + 4

(b) (i) Evaluate $\frac{1}{2\pi i} \oint \frac{e^{tz}}{z^2(z^2+2z+2)} dz$ around $|z| = 3$.

(ii) Find the Laurent series about the indicated singularity for $f(z) = \frac{e^{2z}}{(z-1)^3}$; $z = 1$. 4+4

(c) (i) State and prove Liouvlille's theorem.

(ii) The only singularities of a single valued function $f(z)$ are poles of order 1 and 2 at $z = -1$ and $z = -2$, with residues at these poles 1 and 2 respectively. If $f(0) = 7/4$, $f(1) = 5/2$, determine $f(z)$. 4 + 4

(d) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos^2\theta)}$, $a > b > 0$

(ii) If the mapping $w = f(z)$ is conformal then show that $f(z)$ is an analytic function of z. 4 + 4

(e) Using the method of residues, evaluate $\int_{-\infty}^{\infty} \frac{x\sin\pi x}{x^2+2x+5} dx$. 8

(f) (i) Express the transformation $u = 4x^2 - 8y, v = 8x - 4y^2$ in the form $w = F(z, \bar{z})$.

(ii) Prove that Schwarz-Christoffel transformation maps the upper half plane onto the interior of the polygon. 3+5