2024
SRMUJ
M.Sc.
1st Semester Examination
Mathematics
MTM-105
(Partial Differential Equations and Generalized Functions)

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any four of the following questions: **4×2**

a) Define Dirac-delta function. Find the Fourier transformation of the Dirac-delta function.

b) If $L(u) = Au_{xx} + Bu_{xy} + Cu_{yy} + Du_y + Eu_y + Fu$, then find its adjoint operator L^*, where $A, B, C\ , D, E$ and F are functions of x and y.

c) Solve: $\left(D^3 - 6D^2D' + 11DD'^2 - 6D'^3\right)z = e^{5x+6y}$.

d) Define well-posed mathematical problem with example. Also give an example of an ill-posed problem.

e) Describe 'Spherical mean' of a harmonic function.

f) Construct a PDE from the equation $u = ae^{-b^2t}\cos bx$, where *a* and *b* are arbitrary parameters.

2. Answer any four of the following questions: **8×4**

a) (i) Using D'Alembert's formula, solve the wave equation

$$u_{tt} - u_{xx} = 1, \quad -\infty < x < \infty,\ t > 0$$

Subject to

$$u(x,0) = x^2, \quad -\infty < x < \infty$$
$$u_t(x,0) = 1, \quad -\infty < x < \infty.$$

(ii) Solve the PDE: $\left(x^2D^2 - 2xyDD' + y^2D'^2 - xD + 3yD'\right)z = 8\frac{y}{x}$. 4 + 4

b) Classify and reduce the PDE $u_{xx} - 2\sin x\, u_{xy} - cos^2 x u_{xy} - \cos x\, u_y = 0$ to a canonical form and hence solve it. 8

c) (i) Find the equation of the integral surface of the linear PDE $(x-y)p + (y-z-x)q = z$ which contains the circle $x^2 + y^2 = 1, z = 1$.
(ii) Find the complete integral of $pxy + pq + qy = yz$. 4 + 4

d) (i) A string of length L is released from rest in the position $y = f(x)$. Show that the total energy of the string is

$$\frac{\pi^2 T}{4L}\sum_{n=1}^{\infty} n^2 {k_n}^2$$

where $k_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$ and T is the tension of the string.

(ii) Prove that if the Neumann problem for a bounded region has a solution, then it is either unique or it differs one another by a constant. 5 + 3

e) (i) Obtain the solution of the one-dimensional diffusion equation $u_t = ku_{xx}$, satisfying the following conditions: (i) T is bounded as $t \to \infty$, (ii) $T(0,t) = T(\pi,t)$, $t \geq 0$ (iii) $T(x,0) = f(x) = x, \qquad 0 \leq x \leq \frac{\pi}{2}$

$$= \pi - x, \ {}^{\pi}/_{2} \leq x \leq \pi.$$

(ii) Consider the Cauchy problem for Laplace equation $u_{xx} + u_{yy} = 0$, subject to $u(x,0) = 0$, $u_y(x,0) = \frac{1}{n}\sin nx$, where n is a positive integer. Show that the solution is $u(x,y) = \frac{1}{n^2}\sin nx \sinh ny$. 4 + 4

f) Obtain the Poisson's integral formula of an interior Dirichlet problem for a circle. 8