

**2024**  
**SRMUJ**  
**3rd Semester Examination**  
**M. Sc.**  
**Mathematics**  
**MTM-302**  
**Classical Mechanics and Non – linear Dynamics**

**Full Marks: 40**

**Time: 2 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations used here have their usual meaning.

**1. Answer any four of the following questions: 2 × 4**

- (a) Express Hamilton's equations of motion in terms of Poisson's bracket.
- (b) If the Lagrangian,  $L = L(q_i, \dot{q}_i, t)$ , of a dynamical system does not contain a coordinate  $q_i$ , then show that the corresponding conjugate momentum is conserved.
- (c) State the postulates of special theory of relativity.
- (d) For a system of  $N$  particles, show that the rate of change of angular momentum is equal to the applied torque for a system of particles.
- (e) Find the velocity of an electron whose kinetic energy equals to rest mass energy.
- (f) Using linear stability analysis determine the stability of the fixed points for the system  $\dot{x} = \sin x$ .

**2. Answer any four of the following questions: 8 × 4**

- (a) (i) Define symmetrical top. Show by using the Euler's equations of motion for a rigid body that a symmetrical top can have stable rotation only about the principal axis.  
 (ii) Show that for a Scleronomic system having  $n$  degrees of freedom, the quantity  $E = \sum_{j=1}^n \frac{\partial L}{\partial \dot{q}_j} \dot{q}_j - L$  is a constant of motion. 4 + 4
- (b) (i) Prove that the Poisson bracket of two dynamical variables remains invariant under a canonical transformation.  
 (ii) State Hamilton's principle of least action. Write down its importance. 5 + 3
- (c) (i) Establish Euler-Lagrange equations of motion from Hamilton's principle of least action.  
 (ii) Write down the characteristics of normal modes. 6 + 2
- (d) (i) A mechanical system is described by the Hamiltonian  $H(q, p) = \frac{p^2}{2m} + \frac{1}{2}mw^2q^2$ . Show that as a result of canonical transformation generated by  $F(q, Q) = -\frac{Q}{q}$  the Hamiltonian in the new coordinate  $Q$  and the momentum  $P$  will become  $K = \frac{Q^2 P^4}{2m} + \frac{1}{2}mw^2 P^{-2}$ .

(ii) The Lagrangian of a dynamical system in two-dimension is  $L(x, y, \dot{x}, \dot{y}) = m\dot{x}\dot{y} + \frac{1}{2}m\dot{x}^2$ .

Determine the Hamiltonian.

5 + 3

(e) (i) If a rectangular parallelepiped with its edges  $2a, 2a, 2b$  rotates about its centre of gravity under no forces, prove by using the Euler's equations of motion that its angular velocity about one principal axis is constant and about other axis it is periodic.

(ii) In relativistic mechanics, if  $p$  be the momentum and  $m_0$  be the rest mass of a particle, then show that the total energy  $E$  is given by  $E = \sqrt{m_0^2 c^4 + c^2 p^2}$ , where  $c$  is the speed of light.

5 + 3

(f) (i) Consider the system  $\dot{x} = -x + y^2$ ;  $\dot{y} = -y + x^2$  and the function  $E(x, y) = x^2 + y^2$ . Discuss the stability of the critical point  $(0, 0)$ , using the Liapunov's Direct method.

(ii) Show that the 4-dimensional volume element  $dx dy dz dt$  is invariant under Lorentz's transformation.

5 + 3