2024
SRMUJ
3rd Semester Examination
M. Sc.
Mathematics
MTM-303
Integral transforms and equations

Full marks: 40 **Time: 2 hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations used here have their usual meaning.

1. Answer any four of the following questions: **2 × 4**

(a) Write the shifting property of Laplace Transform of a function.
(b) What is Singular Integral Equation? Give an example.
(c) What is the nature of the integral equation $f(x)=\int_a^b \frac{1}{x(t-a)}\phi(t)dt$.
(d) Define eigen value and eigen vector of an Integral equation.
(e) Let $f(x)$ be a solution of $\int_0^x e^{x-t}f(t)dt = x, x>0$. Then $f(1)$ =?
(f) Define the wavelet function and analyse the parameters involving in it.

2. Answer any four of the following questions: **8 × 4**

(a) Show that the Integral equation $y(x)=f(x)+\frac{1}{\pi}\int_0^{\pi}\sin(x+t)\,y(t)dt$ possess infinite number of solutions when $f(x)=1$ but possess no solution for $f(x)=x$. 8

(b) (i) Show that $\int_0^t\left(\int_0^v f(u)du\right)dv = L^{-1}\left\{\frac{F(P)}{P^2}\right\}$.

(ii) Solve the ODE $y''(t)-3y'(t)+2y(t)=4e^{2t}$ by using Laplace Transform, given $y(0)=-3, y'(0)=5$. 4 + 4

(c) State Bromnich's Integral formula. Find $L^{-1}\{\frac{p}{(p+1)^3(p-1)^2}\}$ by using complex inversion formula. 2 + 6

(d) (i) Using residue theorem find $f(x)$ where Laplace transform $F(p)=\frac{p}{(p-2)(p^2+4)}$.

(ii) Find the resolvent kernel and using this solve the integral equation

$$\phi(x)=x+\int_0^1 (t-x)\phi(t)dt.$$

4 + 4

(e) (i) Find the eigen value and eigen functions of the integral equation

$$\phi(x)=\lambda\int_0^{2\pi} Sin(x+t)\phi(t)dt.$$

(ii) Find the solution of the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ $x > 0,\ t > 0$ which remains bounded for $x \geq 0$ and following initial and bounded conditions $u(x,0) = 0,\ u(0,t) = f(t)$. 4 + 4

(f) State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that $\int_{-\infty}^{\infty} \frac{dx}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} = \frac{\pi}{ab(a+b)}, a, b > 0$. 8