

2025
SRMUJ
4th Semester Examination
M. Sc.
Mathematics
MTM - 401
Continuum Mechanics

Full Marks: 40

Time: 2 hours

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

1. Answer any four of the following questions: 4 × 2

- (a) A velocity field is specified in Lagrangian form: $v_1 = -X_2 e^{-t}$, $v_2 = -X_3$, $v_3 = 2t$. Determine the acceleration components in Eulerian form.
- (b) The motion of a body described by the mapping: $x_1 = X_1 + t^2 X_2$, $x_2 = X_2 + t^2 X_1$, $x_3 = X_3$. Find the position of the particle in undeformed configuration that occupies the position (9, 6, 1) at time $t = 2$.
- (c) Find the relation between α and β such that the small deformation defined by $u_1 = \alpha x_1 + 3x_2$, $u_2 = x_1 - \beta x_2$ and $u_3 = 3x_3$ is isochoric.
- (d) Find the complex potential due to source.
- (e) Evaluate directly all stress invariants for the stress tensor $[T_{ij}] = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
- (f) Show that the difference between two values of a stream function at two points represents the flux of a fluid across any curve joining these two points.

2. Answer any four of the following questions: 4 × 8

- (a) (i) Determine the restriction on f_1, f_2, f_3 if $\frac{x^2}{a^2} f_1(t) + \frac{y^2}{b^2} f_2(t) + \frac{z^2}{c^2} f_3(t) = 1$ is a possible boundary surface of a liquid. 4
- (ii) Given the following Cauchy stress components: $\sigma_{11} = -2x_1^2$, $\sigma_{22} = 3x_1^2 - 2x_2^2 + 5x_3$, $\sigma_{33} = -5 + x_1 + 3x_2$, $\sigma_{12} = -7 + 4x_1 x_2 + x_3$, $\sigma_{23} = 0$, $\sigma_{31} = 1 + x_1 - 3x_2$. Determine the body force components for which the stress field describes a state of equilibrium. 4
- (b) (i) Derive the extensional strain tensor. The strain tensor at a point is given by

$$(E_{ij}) = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Determine the extension of the line element in the direction of (2/3, 2/3, 1/3). 2 + 2

- (ii) What is significance of an image. Find the image of a source with respect to a straight line. 4
- (c) (i) Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two-dimensional fluid. Find the stream lines and path lines. 5
- (ii) Show that in two dimensional irrotational fluid motion, stream function $\psi(x, y)$ and velocity potential $\phi(x, y)$ satisfies Laplace equation. 3
- (d) (i) Show that the following are not possible strain components:
 $e_{11} = k(x_1^2 + x_2^2)$, $e_{12} = k'x_1x_2x_3$, $e_{13} = 0 = e_{23} = e_{33}$, $e_{22} = k(x_2^2 + x_3^2)$. 4
- (ii) Define principal strain. Show that all principal strains are real. 4
- (e) State and prove energy equation of perfect fluid. Give examples of irrotational and rotational flows. 7 + 1
- (f) Define isotropic linear elastic body. Derive equation of motion for an isotropic linear elastic body. Under what conditions this body shows wave equations for displacement? And derive it. 1 + 4 + 3