2025
SRMUJ
4th Semester Examination
M.Sc.
Mathematics
MTM-402

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

Unit-I [Fuzzy Mathematics with Applications]

1. Answer any two of the following questions **2 × 2**

(a) Define fuzz set and its support and hight.

(b) Write the membership function of the trapezoidal fuzzy number (-5, 2, 4, 8) and also draw its diagram.

(c) Define the product of two interval numbers. Hence find the value of $\sqrt{[4, 8]}$.

(d) State Bellman and Zadeh's principle on fuzzy set theory.

2. Answer any two of the following questions **8 × 2**

(a) Using the rules of subtraction of triangular fuzzy numbers show that $[-1,1,3] - [1,3,5] = [-6, -2, 2]$. 8

(b) Show that for interval numbers the distributive laws don't hold in general. Give an example of fuzzy set with membership function which is neither normal nor convex. Evaluate: $2(5,6,8,12) + 3(-1,3,4) - 5[-3,2] + 8$. 3 + 3 + 2

(c) What is α-cut of a fuzzy set? Show that law of contradiction and law of excluded middle don't hold in fuzzy set in general. 2 + 6

(d) Describe the Werner's method for solving the following fuzzy LPP.

$$\begin{aligned} &\text{Maximize } z = cx \\ &\text{subject to } (Ax)_i \lesssim b_i,\ i = 1, 2, \dots, m \\ &x_j \geq 0. \end{aligned}$$

Describe the Werner's method for solving the following fuzzy LPP.

$$\begin{aligned} &\text{Maximize } z = 5x_1 + 3x_2 + 4x_3 + 9x_4 \\ &\text{subject to } 2x_1 + 2x_2 + 3x_3 + x_4 \leq \widetilde{15} \\ &x_1 - 5x_2 + 3x_3 + 2x_4 \leq \widetilde{40} \\ &2x_1 + 6x_2 - 3x_3 + 9x_4 \leq \widetilde{60} \\ &x_j \geq 0;\ j = 1, 2, 3, 4. \end{aligned}$$

Assuming the tolerance as 5, 10, 20. 8

Unit – II [Real Analysis – II]

3. Answer any two of the following questions: **2 × 2**

(a) Show that every constant function on $[a, b]$ is Lebesgue integrable.

(b) Establish by an example that Monotone convergence theorem need not hold for decreasing sequences of functions.

(c) Using the Lebesgue Dominated Convergence theorem, find the value of the limit and justify your answer:

$$\lim_{n\to\infty} \int_1^{\infty} \frac{\ln(nx)}{x + x^2 \ln(n)} dx$$

(d) State Egoroff's theorem. Write down its significance.

4. Answer any two of the following questions: **2 × 8**

(a) (i) Show that every bounded Riemann integrable function is Lebesgue integrable and the two integrals are equal in this case. 5

(ii) For $n \in \{1, 2, 3, \cdots\}$, let

$$f_n(x) = \begin{cases} 2n, \text{ for } x \in (1/2n, 1/n) \\ 0, \text{ for } x \in (0, 1/2n) \cup (1/n, 1) \end{cases}$$

Calculate $\int_0^1 \lim_{n\to\infty} f_n(x) dx$ and $\lim_{n\to\infty} \int_0^1 f_n(x) dx$. Show also that Fatou's Lemma holds but that Lebesgue dominated convergence theorem does not in this case. 3

(b) Prove that if the bounded functions f and g are measurable on E and are equivalent on E, then $\int_E f(x) dx = \int_E g(x) dx$. Is the converse true? Justify your answer. 5 + 3

(c) (i) Define general Lebesgue integration. Using this definition, show that for two real valued functions f and g, integrable over a measurable set E, the function $f + g$ is integrable and $\int_E (f + g) dx = \int_E f\, dx + \int_E g\, dx$. 1 + 3

(ii) Let $X = Y = [0, 1], \mathcal{A} = \mathcal{B} = \mathcal{B}_{[0,1]}$, and let $\mu = \nu$ be the Lebesgue measure on [0,1]. Define for $x, y \in [0, 1]$, $f(x, y) = \begin{cases} 1 \; if \; x \; is \; rational \\ 2y \; if \; y \; is \; irrational. \end{cases}$

Compute $\int_0^1 (\int_0^1 f(x, y) d\nu(y))\, d\mu(x)$ and $\int_0^1 (\int_0^1 f(x, y) d\mu(x))\, d\nu(y)$. Is $f \in L^1(\mu \times \nu)$? 4

(d) (i) State and prove classical Lebesgue dominated convergence theorem. 1 + 5

(ii) Define $f_n(x)$ on $[0,1]$ as follows:

$$f_n(x) = \begin{cases} n, \text{ if } x \in (0, 1/n) \\ 0, \text{ if } x \notin (0, 1/n) \end{cases}$$

Is $\lim_{n\to\infty} f_n(x)$ exists? Check whether the values of $\lim_{n\to\infty} \int_E f_n(x)\, dx$ and $\int_E \lim_{n\to\infty} f_n(x)\, dx$ are equal or not? 2