2025
SRMUJ
4th Semester Examination
M. Sc.
Mathematics
MTM - 403

Full Marks: 40 **Time: 2 hours**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

Unit – I [Stochastic Process and Regression]

1. Answer any two of the following questions: **2 × 2**

(a) Show by an example, that the Markov chain can be represented by a graph.

(b) State Gambler's ruin problem and write the transition matrix for it.

(c) State strong Markov property.

(d) Define multiple correlation coefficient and partial correlation coefficient.

2. Answer any two of the following questions: **8 × 2**

(a) Define Wiener process. Establish the forward diffusion equation of the Wiener process. Hence find the backward diffusion equation of the Wiener process. 2 + 6

(b) (i) State and prove first entrance theorem.

(ii) Prove that in an irreducible Markov chain all the states are of the same type. They are either all transient, all persistent null, or all persistent non-null. Also, show that all the states are aperiodic and in latter case they all have same period. 4 + 4

(c) (i) Consider the Markov Chain with a transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Test whether the states are periodic and persistent.

(ii) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the digit leaving the nth stage of the system and X_0 be the digit entering the first stage (leaving the 0th stage). At each stage, there is a constant probability q that the digits which enter will be transmitted unchanged (i.e., the digit will remain unchanged when it leaves), and probability p otherwise (i.e., the digit changes when it leaves), $p + q = 1$. Find the one step transition matrix P and n-step transition matrix P_n . Also, find P_n when $n \to \infty$. 4 + 4

(d) Define partial correlation. Prove that $r_{1.23\ldots p} = \left(1 - \frac{|R|}{R_{11}}\right)^{1/2}$, where the symbols have their usual meaning. Hence discuss the case if we take three variables. 2 + 6

Unit – II [Graph Theory]

3. Answer any two of the following questions: **2 × 2**

(a) Let G be a graph with 6 components and 20 edges. Find the maximum possible number of vertices in G.

(b) Is it possible to draw a graph with 4 edges and 4 vertices with degree sequences 1, 2, 3, 4? Explain.

(c) Find the digraph whose adjacency matrix is given below

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

(d) Suppose that a connected, simple planar graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph divide the plane?

4. Answer any two of the following questions: **2 × 8**

(a) (i) Let G be a simple connected graph with n ≥ 3 vertices and e > $\binom{n}{2}$ +1 edges. Show that the Hamiltonian closure of G is complete. 4

(ii) Define the chromatic number χ(G) of a graph. Prove that χ(G) ≤ Δ(G)+1, where Δ(G) denotes the maximum of all degrees of vertices of G. 1 + 3

(b) (i) Show that the chromatic polynomial of any cycle C_n of length n is $P_n(x) = (x-1)^n + (-1)^n(x-1)$. 4

(ii) State Konigsberg Bridge Problem and write a brief note about its solution. 4

(c) (i) What is a perfect matching? Show that a regular bipartite graph has a perfect matching. 1 + 3

(ii) Let G = (V, E) be a connected graph with n vertices, where n ≥ 2. Then show that G has at least two vertices which are not cut vertices. 4

(d) (i) If a simple regular graph has n vertices and 24 edges, then by using the fundamental theorem of Graph theory, find all possible values of n. 3

(ii) A postman has to start at A, walk along all 13 streets and return to A. The numbers on each edge represent the length, in meters, of each street. Find a train that use all the edges of a graph with minimum length. 5