2025
SRMUJ
4th Semester Examination
M.Sc.
Mathematics
MTM-405B
Special Paper: Advanced Operational Research-II

Full Marks: 20 **Time: 1 Hour**

The figures in the margin indicate full marks. Candidates are required to give their answers as far as practicable. Notations have their usual meaning.

1. Answer any two of the following questions: **2×2**

a) For an equipment, the reliability per 100 hours of operation has been estimated to be 0.999. Calculate the failure rate of the equipment.

b) What do you mean by prefix-free encoding? What is the importance of this encoding?

c) Write the extremal property of Entropy function.

d) What are the differences between the reliability of a device and a system of components?

2. Answer any four of the following questions: **2×8**

a) State Euler-Lagrange equation. Find the least value of $\int_A^B \frac{1}{y}[1+(\frac{dy}{dx})^2]^{1/2}\text{dx}$ where A is $(-1,1)$ and B is $(1,1)$. 2 + 6

b) If H denote the entropy function, then prove that $H(q_1, q_2, \dots, q_{n-1}, p_1, \dots, p_m) = H(q_1, q_2, \dots, q_{n-1}) + q_n H\left(\frac{p_1}{q_n}, \frac{p_2}{q_n}, \dots, \frac{p_m}{q_n}\right)$ where $q_n = \sum_{i=1}^{m} p_i$. 8

c) (i) In a system, there are n number of components connected in parallel with reliability $Ri(t);\ i = 1, 2, \dots, n$. Find the reliability of the system. If $R_1(t) = R_2(t) = \cdots = R_n(t) = e^{\lambda t}$, then what will be the expression of system reliability?

(ii) Define system Reliability. Find the reliability of a system with two components of which one is a stand-by. The components are connected in parallel. 4 + 4

d) Define marginal and conditional entropies. A transmitter has a character consisting of five letters $(x_1,\ x_2,\ x_3,\ x_4,\ x_5)$ and the receiver has a character consisting of four letters $(y_1,\ y_2,\ y_3,\ y_4)$. The joint probability for the communication is given below:

$P(x_i, y_j)$	y_1	y_2	y_3	y_4
x_1	0.25	0	0	0
x_2	0.10	0.30	0	0
x_3	0	0.05	0.10	0
x_4	0	0	0.05	0.10
x_5	0	0	0.05	0

Determine $H(X)$, $H(Y)$, $H(X, Y)$ and $H(Y = X)$. 8